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TRANSFORMATION OF RICE'S SMALL PERTURBATION RESULTS FOR ROUGH SURFACE SCATTERING INTO A COMPREHENSIVE SINGLE SCATTER SOLUTION THAT INCLUDES PHYSICAL AND GEOMETRICAL OPTICS

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ABSTRACT. *Using a straightforward two-step procedure, Rice's original small perturbation solution to rough surface scattering is transformed into a comprehensive single scatter solution, that includes the small perturbation and the physical/geometrical optics solutions. The first involves a phase restoration and the second involves subjecting the surface element scattering coefficients to the principle of invariance to coordinate transformations.*

1. INTRODUCTION

To obtain the rough surface like and cross polarized scattering coefficients, Rice [1] adopted the Rayleigh hypothesis (associated with upward scattered waves from slightly rough surfaces), expanded the phase term associated with the height fluctuations in an infinite Taylor series and imposed approximate boundary conditions for rough surfaces with very small slopes. However, Rice's solutions are not in agreement with the solutions based on the Kirchhoff approximations for the surface fields at the rough interface (even when the radius of curvature is very large and root mean square height of the surface is very small compared to the electromagnetic wavelength) especially as the root mean square slopes become smaller and smaller. In fact, the small perturbation series solution exhibits a rather peculiar behavior. The higher order perturbation terms become larger than the lower order terms as the root mean square slope decreases [2,3]. The problems associated with convergence were first raised by Rice himself in his original monumental contribution [1]. In it, he states "In this work, we shall overlook questions of convergence although they may perhaps be treated by placing suitable restrictions on the (Fourier) components of the surface."

Numerous attempts have been made in order to broaden the class of rough surface problems that can be solved using Rice's basic results. These include adding higher order perturbation terms as well as the adoption of a hybrid perturbed - physical optics method based on two scale models of the rough surface.

It is shown, using a straightforward two step procedure, that Rice's original solution can be transformed into a very comprehensive single scatter solution that includes the small perturbation and the physical/geometrical optics solutions based on the Kirchhoff approximation for the surface fields [4]. The first involves a phase restoration. Thus the height fluctuation about the mean plane appears in the phase of the scattered field rather than as a coefficient. The second involves subjecting the surface element scattering coefficients to the principle of invariance to coordinate transformations.

Thus Rice's transformed solutions are in complete agreement with the single scatter version of the full wave solutions [3,5] and they reduce to the corresponding physical optics solution when the normal to the rough surface $\bar{n}$ at any point is replaced by its value at the stationary phase points.

2. RICE'S SOLUTION FOR THE DIFFUSELY SCATTERED ELECTROMAGNETIC FIELDS

Rice's small perturbation solution [1] for the electromagnetic fields diffusely scattered from slightly rough surfaces $y_s = h(x_s, z_s)$ (small slopes and small height fluctuations compared to wavelength) can be expressed in matrix form as follows:

$$G_s^f = G_0 \int_{-\ell}^{\ell}\int_{-L}^{L} S(\bar{k}^f, \bar{k}^i)\exp(i\bar{v}\cdot\bar{r}_t)[iv_y h(x_s, z_s)]dx_s dz_s G^i \tag{1}$$

in which G^f and G^i are 2×1 matrices whose elements are the vertically and horizontally polarized components of the scattered and incident fields E^{Pf} and E^{Pi} (P = V,H), respectively. The coefficient G_0 is

$$G_0 = k_0^2 \exp(-ik_o r)/2\pi i v_y r \tag{2}$$

where k_0 is the free space wave number, r is the distance from the origin (on the mean plane y = 0) to the observation point and

$$\bar{v} = \bar{k}_0^f - \bar{k}_0^i = k_0(\bar{n}^f - \bar{n}^i) = v_x\bar{a}_x + v_y\bar{a}_y + v_z\bar{a}_z = \bar{v}_t + v_y\bar{a}_y \tag{3}$$

in which $\bar{n}^i$ and $\bar{n}^f$ are unit vectors in the direction of the incident and scattered waves. The position vector to a point on the rough surface is $\bar{r}_s = x_s\bar{a}_x + h\bar{a}_y + z_s\bar{a}_z = \bar{r}_t + h\bar{a}_y$. The 2×2 scattering matrix S is given by [5,6,7]

$$S(\bar{k}^f, \bar{k}^i) = 2\cos\theta_0^f \cos\theta_0^i R(\bar{k}^f, \bar{k}^i) \tag{4}$$

and the elements of the 2×2 matrix R are

$$R^{VV} = \frac{[\mu_r C_1^f C_1^i \cos(\phi^f - \phi^i) - S_0^f S_0^i](1 - 1/\epsilon_r) + (1 - \mu_r)\cos(\phi^f - \phi^i)}{(C_0^f + \eta_r C_1^f)(C_0^i + \eta_r C_1^i)} \tag{5}$$

$$R^{HH} = \frac{[\epsilon_r C_1^f C_1^i \cos(\phi^f - \phi^i) - S_0^f S_0^i](1 - 1/\mu_r) + (1 - \epsilon_r)\cos(\phi^f - \phi^i)}{(C_0^f + C_1^f/\eta_r)(C_0^i + C_1^i/\eta_r)} \tag{6}$$

$$R^{HV} = \frac{-\sin(\phi^f - \phi^i)n_r[(1 - 1/\mu_r)C_1^f - (1 - 1/\epsilon_r)C_1^i]}{(C_0^f + C_1^f/\eta_r)(C_0^i + \eta_r C_1^i)} \tag{7}$$

$$R^{VH} = \frac{\sin(\phi^f - \phi^i)n_r[(1 - 1/\epsilon_r)C_1^f - (1 - 1/\mu_r)C_1^i]}{(C_0^f + \eta_r C_1^f)(C_0^i + C_1^i/\eta_r)} \tag{8}$$

in which ϵ_r, μ_r, η_r and n_r are the relative permittivity, permeability, intrinsic impedance and refractive index for medium below the rough interface $y_s < h(x_s, z_s)$. The height fluctuation in (1) results in a fluctuation of the field intensity. This is because on imposing the boundary conditions assuming that $k_0 h \ll 1$, Rice approximates $\exp(iv_y h)$ by $1 + iv_y h$. Thus, the small height approximation is removed by replacing $\exp(i\bar{v}\cdot\bar{r}_t)(iv_y h)$ in (1) by

$$[\exp(i\bar{v}\cdot\bar{r}_s) - \exp(i\bar{v}\cdot\bar{r}_t)] \tag{9}$$

and the height fluctuation results in the fluctuation of the phase and not the field intensity. To remove the small slope approximation that Rice assumes on imposing the boundary condition,

the rough surface element (differential) scattering matrix (4) is subjected the principle of invariance to coordinate transformations. Thus in (1) the projection $dx_s dz_s$ of the rough surface element at $\bar{r}_s$ is replaced by the surface element area $dx_s dz_s/(\bar{n}\cdot\bar{a}_y)$ (where $\bar{n}$ is the unit vector normal to the surface). Furthermore in (5)-(8) the expressions for the sines and cosines of the angles of incidence and scatter for free space, $S_0^i = \sin\theta_0^i$, $S_0^f = \sin\theta_0^f$, $C_0^i = \cos\theta_0^i$, $C_0^f = \cos\theta_0^f$ as well as the related expression with subscripts 1 for the medium $y < h$ are replaced by the corresponding expressions involving the local angels of incidence and scatter θ_0^{in} and θ_0^{fn} [5]. Furthermore, the fixed planes of incidence and scatter (normal to $\bar{n}^i \times \bar{a}_y$ and $\bar{n}^f \times \bar{a}_y$) are replaced by the local planes of incidence and scatter (normal to $\bar{n}^i \times \bar{n}$ and $\bar{n}^f \times \bar{n}$) through the transformation matrices T^f and T^i. Thus $S(\bar{k}^f,\bar{k}^i)$ in (4) is replaced by [5]

$$S(\bar{k}^f,\bar{k}^i) \implies T^f S_n(\bar{k}^f,\bar{k}^i)T^i = D(\bar{n}^f,\bar{n}^i) \tag{10}$$

These two steps transfer Rice's small perturbation solution into the full wave solution for the single scattered far fields [3,5]

$$G_s^f = G_0 \int_{-\ell}^{\ell}\int_{-L}^{L} D(\bar{k}^f,\bar{k}^i)[\exp(iv\cdot\bar{r}_s) - \exp(i\bar{v}\cdot\bar{r}_t)]\frac{dx_s dz_s}{\bar{n}\cdot\bar{a}_y}G^i \tag{11}$$

It is shown that when the correlation between the surface heights and slopes are accounted for, in the analysis in the high frequency limit (11) reduces to the physical optics solution with $D(\bar{k}^f,\bar{k}^i)$ evaluated at the stationary phase (specular) points where $\bar{n} \to \bar{n}_s = \bar{v}/v$ [3]. Note that at these stationary phase points R^{VV} (5) and R^{HH} (6) reduce to the familiar Fresnell reflection coefficients at the specular points while R^{HV} and R^{VH} vanish at these points.

The relationships between the full wave solution, Rice's small perturbation solution, the physical/geometrical optics solution and the hybrid solution based on the two scale model have been discussed in some detail in a recent publication [8].

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